JOURNAL OF FINANCIAL INTERMEDIATION 4, 358–395 (1995)

Convertible Debt as Delayed Equity: Forced versus Voluntary Conversion and the Information Role of Call Policy

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Received April 26, 1994

There is a common perception that many firms issue convertible debt as a form of delayed equity. The advantage of issuing equity in this delayed manner has been linked to the lower adverse selection properties of convertible debt as compared to equity; one can first issue convertibles and later call, forcing conversion, presumably preserving the initial advantage of the convertibles. However, this paper suggests that the benefits of callable convertible debt as delayed equity are preserved only if conversion is voluntary. This appears to be consistent with the empirical evidence. *Journal of Economic Literature* Classification Numbers: G32, D82. © 1995 Academic Press, Inc.

Convertible securities, in many cases, are employed as deferred common-stock financing. Technically these securities represent debt or preferred stock, but in essence they are delayed common stock.
—James van Horne, "Financial Management and Policy"

1. INTRODUCTION

This paper addresses the issue of callable convertible debt as delayed equity, emphasizing the issue of forced versus voluntary conversion. Pilcher (1955), Brigham (1966), and Hoffmeister (1977) confirm the view that a

* I thank Anat Admati, Dick Brealey, Michael Brennan, Julian Franks, Michel Habib, Charles Jacklin, Ernst Maug, Paul Pfleiderer, David Pyle, and Steve Umlauf for comments. I also thank two anonymous referees and seminar participants at Baruch, CUBS, Gothenburgh, LBS, LSE, Strathclyde, the EFA in Brussels 1994, and the Corporate Finance Symposium at Odense University, May 1995.

1042-9573/95 $12.00

large proportion of firms issue convertible debt as an indirect means to add equity to the capital structure. Brigham, for example, reports that 68% of firms viewed their convertible issues as delayed equity. Issuing convertibles was viewed as preferable to issuing equity outright because managers "... believed [their] stock's price would rise over time, and convertibles provide a way of selling common stock at a price above the existing market." (Brigham, 1966, p. 51).

More recently, Stein (1992) has argued that the advantage of callable convertibles as delayed equity arises from information asymmetry between managers and investors: "When used with a call provision that enables early forced conversion, a convertible can serve as an indirect ... mechanism for implementing equity financing that entails less of an adverse price impact than an offering of common stock." (Stein, 1992, p. 19). In support of Stein's hypothesis, the empirical evidence summarized by Smith (1986) reports that the average abnormal return (AAR) associated with an issue of equity is −3.14% versus −2.07% for convertible debt.

Given that issuing convertible debt has a less negative effect on the stock price than issuing equity, the question arises as to why all firms that wish to raise equity capital do not do so by first issuing convertibles and later force conversion. Stein's suggested answer is that not all firms are likely to experience the initial increase in share price required for the conversion value of the convertibles to rise above the call price. For such firms, forced conversion is an unlikely option, and they do not issue convertible debt because of the high probability of deteriorating to the point of costly financial distress. However, the empirical evidence suggests that this is only half the story.

Firms that issue callable convertibles and later force conversion experience an adverse price impact twice—first, when they issue convertibles and second, when they make a conversion-forcing call. Mikkelson (1981) finds an AAR of −2.08% around the announcement date of calls that force conversion. Thus, the evidence suggests that issuing equity by first issuing convertibles and later forcing conversion leads to a larger negative abnormal return (−4.15%) than issuing equity outright (−3.14%). This evidence does not constitute a direct test of the hypothesis that issuing callable convertibles and later forcing conversion involves a greater adverse share price impact than issuing equity outright. But it is suggestive, especially since there is no proper test of this proposition in the literature.

I develop a model which yields this proposition as a prediction; the benefits of convertible debt as delayed equity are preserved only if conversion is voluntary, either at maturity, or, perhaps, concurrent with a dividend increase. This contrasts with Stein's model, which is inconsistent with the empirical evidence as it does not predict abnormal returns around conversion forcing calls. Stein's model does not capture the distinction between forced and voluntary conversion. He argues that it is the possibility of forced conversion that makes callable convertibles preferable to outright equity issues for some firms.

Like Stein's, my model is based on asymmetric information between management and investors. Consistent with the empirical evidence (see Smith, 1986), in both models, firms of the highest quality issue straight debt, firms of intermediate quality issue convertible debt, and firms of the lowest quality issue equity. However, our models differ in the treatment of call policy. In my model, calls convey information to investors in addition to the information transmitted at the issue date; in Stein's model calls provide no additional information. This gives rise to differing rationales for why low-quality firms prefer to issue equity rather than convertibles even though the latter results in a less adverse stock price reaction. In Stein's model, low-quality firms are deterred from issuing convertible debt by high bankruptcy costs. In my model, the role of equity as a "safety valve" is pushed forward into the call policy; while all firms may be in a situation where they can force conversion in the future, low-quality firms are more likely to do so. But forced conversion would wipe out the initial gains from issuing convertible debt. The focus is thus on the information role of call policy, rather than on financial distress per se, as a deterrent to issuing convertible debt.[1]

If it is true that the benefits of convertible debt as delayed equity are preserved only if conversion is voluntary rather than forced, one would expect to observe firms to delay calling outstanding convertibles rather than force conversion at the first opportunity. This is in fact what empirical research has found.[2] Ingersoll (1977b) finds that in his sample the median firm delayed calling until the conversion value exceeded the call price by 43.9%.

Asquith and Mullins (1991) also document that firms typically delay forced conversion, but emphasize cash flow incentives rather than information effects. For a large proportion of firms with outstanding convertible debt in their sample, the aftertax coupon is often lower than the dividend that would be paid if the convertibles were converted (ignoring the possibility of simultaneously forcing conversion and reducing dividends). Dividend

[1] As in Stein's paper, the decision to issue equity in my model can be viewed as driven by costs imposed upon management in financial distress. The difference is that in my paper this cost would be felt indirectly, through the call policy. Although I have stressed the cost of deterioration as a cost of financial distress, it could also be management losing their jobs, bonus, etc. if the stock price declines "too much" (bankruptcy is not necessary, as will be discussed in footnote 11).

[2] This contrasts with what would be predicted by perfect markets theory. Brennan and Schwartz (1977) and Ingersoll (1977a) show that in perfect markets share value is maximized if convertibles are called as soon as their value, if not called, is equal to the call price.

signalling theory (e.g., Miller and Rock, 1985) suggests that a call in these circumstances should be good news since net payout would be increased. Surprisingly then, Campbell *et al.* (1991) find statistically insignificant abnormal returns around this event. The current model sheds light on this puzzling result by suggesting that the negative news associated with extinguishing the put option element of the convertible negates the good news associated with increasing net payout.

My model incorporates Harris and Raviv's (1985) well-known model of signalling through convertible debt call policy. In that model, a risk-averse manager must decide whether to call an outstanding issue of convertibles. While the decision to issue callable convertible debt is not endogenous in Harris and Raviv's analysis, I show that callable convertibles can be issued for informational reasons, serving as an intermediate signal between equity and straight debt. I also argue that callable convertible debt is preferred to the noncallable variant. This conclusion would not be changed by the possibility of a swap of noncallable convertible debt into equity. Essentially, the advantage of callable convertible debt is that it provides insurance against adverse stock price movements in the future. This is valuable to risk-averse managers.

While new equity would be even better in terms of its insurance properties, the disadvantage of equity stems from a greater adverse selection problem (see, e.g., Myers and Majluf, 1984). This tradeoff is not dependent, however, on the manager being risk-averse. The tradeoff between insurance and adverse selection is the driving force in more traditional models such as Stein's where the risk-neutral manager's objective is to maximize the true value of existing shares less the expected value of his private cost of bankruptcy.[3] Insurance against bankruptcy would be valuable to this manager; under symmetric information, he would prefer issuing equity to any kind of risky debt. The private bankruptcy costs induce a nonlinearity in the manager's objective function which manifests itself in risk-averse-like behavior. The tradeoff between insurance and adverse selection is an intuitive way of looking at the choice of financing, which is not contingent on the model-specific details of having a risk-averse manager (as in my model) or private bankruptcy costs (as in Stein's model).

Viewed in this light, my model differs from Stein's in the source of the cost of insurance with callable convertible bonds. In my model, the cost is endogenously derived from a fall in the share price when a firm forces conversion. In other words, making use of the insurance provided by the call feature is costly. In fact, it can be sufficiently costly to deter lower quality firms from issuing callable convertibles. In Stein's model, the cost is exogenously derived from a high probability of not being in a position

[3] See also Ross (1977). The insurance versus adverse selection tradeoff is also apparent in Leland and Pyle (1977) and has been explicitly modelled by Rothschild and Stiglitz (1976).

in the future to force conversion. In reality, both of these effects may be important deterrents to issuing callable convertible debt. However, as discussed above, Stein's explanation is not by itself consistent with the empirical evidence. The effect I focus on more completely explains the empirical evidence.[4]

Previous theoretical work on financing with convertible debt has focused on noncallable convertibles. Green (1984) shows that convertible debt provides a solution to the moral hazard problem of straight debt. Brennan and Kraus (1987) and Constantinides and Grundy (1989) develop models of costless signalling with noncallable convertible debt. However, the vast majority of convertibles issued in the United States are callable; out of a total of 491 U.S. convertibles listed in *Moody's Bond Record* (July 1994), only 25 are noncallable.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 provides analysis and discusses noncallable convertible debt and call protection. Section 4 derives predictions. In addition to the listed predictions, all of Harris and Raviv's are retained. The robustness of the separating equilibrium giving rise to the predictions is examined in Section 5. Section 6 provides discussion on dividend and call policy and offers some concluding remarks.

2. THE MODEL

There are four dates, $t = 0, 1, 2, 3$. Figure 1 is a sketch of the key items per date. At time 0, an all equity, nondividend paying firm has an investment opportunity which costs I. The firm has no assets in place and will have cash flows of zero at all dates if it does not invest. If the firm invests, it will have uncertain cash flows $\tilde{\delta}$ at date 3 and zero at other dates. The realization of $\tilde{\delta}$ is either H (high) or L (low), $I < L < H$.

The firm lacks internal funds. The manager must, therefore, raise the cost of investment by either issuing convertible debt ($\mathcal{CD}$), straight debt ($\mathcal{SD}$), or equity ($\mathcal{E}$). Debt matures at date 3. If the firm issues equity or straight debt, the manager has no further decisions to make.

If convertible debt is issued, the *conversion game* takes place:[5] The manager must decide at dates 1 and 2 whether to call the convertibles. If the

[4] In the Harris and Raviv model, there are numerous equilibria. In this paper, I provide conditions under which their separating equilibrium is unique when applying the Cho and Kreps (1987) *intuitive criterion.* I also show that when the choice of financing is endogenous, similar conditions will leave the separating equilibrium unique when applying Banks and Sobel's (1987) *divinity* refinement.

[5] This follows Harris and Raviv (1985). The role of date 2 is to analyze the incentives for delaying forced conversion. Call protection periods are discussed in a later subsection.

$t = 0$ i. Manager receives private information ϕ_0 about the payoff of his project.
ii. Manager raises an amount I by selling one of: straight debt, convertible debt, or equity. Manager invests I in his project.

$t = 1$ i. Manager receives updated private information $\tilde{\phi}_1$.
ii. If convertibles issued at $t = 0$, manager decides whether to call the debt.

$t = 2$ i. Manager receives updated private information $\tilde{\phi}_2$.
ii. If convertibles issued at $t = 0$ and not called at $t = 1$, manager decides whether to call the debt.

$t = 3$ i. Uncertainty is resolved.
ii. If convertibles have not been called, investors decide whether to convert.
iii. Securities are repaid.

FIG 1. Sketch of model: Information and decisions.

manager calls, convertible bond holders will convert provided that the conversion value is larger than the call price. At date 3, assuming that the convertibles cannot be called is equivalent to assuming that the call price is equal to the face value of the convertibles. For convenience of notation, I assume the former. Voluntary conversion is not an issue before date 3 because the firm does not pay dividends. If the manager has not called by period 3, the bondholders convert if and only if the payoff from converting is larger than the face value.

At $t = 0$, the manager receives proprietary information about the likelihood of a high payoff. At $t = 1, 2$, he receives updated proprietary information. Investors do not receive any exogenous information, but they make inferences from the manager's decisions. At date 3 uncertainty is resolved and all securities are paid off. The equilibrium concepts employed in this paper are sequential equilibrium (Kreps and Wilson, 1982) and its refinements.

The manager seeks to maximize expected utility. His utility function has the form: $W = w(S_0) + w(S_1) + w(S_2) + w(S_3)$, where S_t is the firm's stock price at date t and $w(\cdot)$ is concave and strictly increasing. This can be motivated by thinking of the manager as an entrepreneur who owns some shares in the firm and finances consumption by selling shares each period.[6]

[6] This can be viewed as an assumption that the manager smooths consumption over time, which is consistent with risk aversion. One could also motivate the stated objective function by thinking of the manager's wage each period as a function of the stock price. If only a fraction of his wage is determined by the stock price, his objective function may still resemble the one I have assumed. Other models where the manager seeks to maximize a function of several periods' share prices includes Miller and Rock (1985). Like that model, signalling in the current model works because the manager has an interest in how his current actions affect the future stock price. For the broad results to go through, it is important that the manager has an interest in each period's stock price, and that low stock prices are in some sense punished. The role of risk aversion in the model is quite similar to that played by bankruptcy costs in Stein's (1992).

The risk-free rate is normalized to zero, and investors are assumed to price all securities at their expected value conditional on all relevant information. As a result, since $I < L$, the rate of return on the project is higher than the risk-free rate with certainty. The project will, therefore, be financed. The issue is how.

Financing Choices

Initially, the firm has only N shares outstanding. No other securities have been issued. There are three different ways in which the project can be financed. The firm's shareprice may depend on the choice of financing, but the new securities will have a market value of I.

1. Issue Q new shares.

2. Issue zero-coupon straight debt. Since the riskfree rate is normalized to 0 and since the rate of return on the project is higher than the risk-free rate with certainty, the total face value must be equal to I.

3. Issue zero-coupon callable convertible debt. Here the important parameters are the total face value and the fraction of all shares that convertible bondholders will own upon conversion, i.e., the *dilution*. The total face value of the convertibles will be denoted by M. Without loss of generality, it will be assumed that the face value of an individual convertible bond is 1 so that the total face value is also equal to the number of convertible bonds. Denoting the conversion ratio of 1 convertible by r, the total dilution of the convertibles is given by $rM/(N + Mr)$. On a per convertible basis, the dilution is thus $r/(N + Mr)$.

It will be assumed that the total value of the convertibles, I, is greater than the straight debt value of the convertibles, M. This is a realistic assumption which streamlines the analysis somewhat without affecting the main conclusions.[7]

It is assumed that the call price is set such that the firm can force conversion; specifically, the call price is assumed equal to zero. This allows the analysis to focus on the informational role of call policy as a deterrent to

[7] It will be pointed out where this assumption is used. That convertible bonds are typically issued at a premium over their straight debt value (convertible bonds are typically issued with lower coupons than comparable straight bonds) adds empirical support to the assumption that $M < I$. One can view this as roughly meaning that a convertible bondholder's right to convert is at least as valuable as the manager's right to call. Under complete information, the value of convertible bonds *would* be greater than their straight debt value since, as is well known, a (noncallable) convertible bond is equivalent to a straight bond plus a warrant.

issuing convertible bonds and on the issue of forced versus voluntary conversion.[8]

To summarize, the endogenous parameters are r (conversion ratio) and Q (number of new shares); the exogenous parameters are H (high payoff), L (low payoff), N (number of old shares), and I (cost of investment and straight-debt face value). The parameter M (face value of convertible debt and number of convertible bonds) is endogenous in that it determines the market value of the convertibles jointly with the conversion ratio. However, in the proof of the existence of a separating equilibrium (Theorem 1), the approach will be to fix M exogenously and let the conversion ratio be determined in equilibrium by the financing constraint (the value of the issued securities must be equal to I).

Date 3 Share Price

Because uncertainty is resolved at date 3, the share price at this date plays a prominent role in the analysis. The share price is a function of the manager's decisions at dates 0, 1, and 2 and of the realization of $\tilde{\delta}$; i.e., S_3 can be written $S_3(d_0d_1d_2\delta)$, where d_t is the manager's decision at time t. Convertible bondholders' decision to convert voluntarily at date 3—if conversion has not been forced earlier—involves no strategic considerations and can be factored into the determination of S_3. The convertibles are converted voluntarily in period 3 only if the conversion value is larger than the face value, i.e., $\delta r/(N + Mr) > 1$.

If d_0 is $\mathscr{CD}$, d_1 and d_2 are either C (call) or N (not call). If $d_0 = \mathscr{CD}$ and $d_1 = C$, the manager has no further decisions to make. This also applies if d_0 is $\mathscr{SD}$ or $\mathscr{E}$. In these cases, the manager's "decision" is written as *NoAction*.

Date 3 share prices given the total return on the project and the manager's set of decisions are listed in Table I. If convertible debt is issued and converted, the date 3 share price is $\delta/(N + Mr)$.

Prior Probabilities and the Manager's Proprietary Information

The manager has private information regarding the prospects of the investment opportunity, $\tilde{\phi}_0$, at date 0 with a value of G_0 (good), B_0 (bad), or A_0 (awful). The manager receives more private information at dates $t =$ 1, 2, denoted $\tilde{\phi}_1$, $\tilde{\phi}_2$ with values B_t or G_t. Based on his information, the manager develops forecasts by assessing the probability of a high or low payoff. For example, the conditional probability of a high payoff is denoted

[8] However, convertible bonds are typically issued with a positive call price, often equal to face value plus accrued and unpaid interest. It is also common for the call price in early years to be at a premium over the face value, but to fall to the face value at, or some time prior to, maturity. Adapting the model to having a positive call price is discussed in Section 3.3.

TABLE I
DATE 3 SHARE PRICE AS A FUNCTION OF THE MANAGER'S DECISIONS

Decisions			Date 3 Share Price	
d_0	d_1	d_2	if $\tilde{\delta} = H$	if $\tilde{\delta} = L$
$\mathcal{SD}$	—	—	$\frac{H - I}{N}$ ($h_{\mathcal{SD}}$)	$\frac{L - I}{N}$ ($l_{\mathcal{SD}}$)
$\mathcal{E}$	—	—	$\frac{H}{N + Q}$ ($h_{\mathcal{E}}$)	$\frac{L}{N + Q}$ ($l_{\mathcal{E}}$)
$\mathcal{CD}$	C	—	$\frac{H}{N + Mr}$ ($h_{\mathcal{CD}}$)	$\frac{L}{N + Mr}$ ($l_{\mathcal{CD}}$)
$\mathcal{CD}$	N	C	$\frac{H}{N + Mr}$ ($h_{\mathcal{CD}}$)	$\frac{L}{N + Mr}$ ($l_{\mathcal{CD}}$)
$\mathcal{CD}$	N	N	$\min\left\{\frac{H}{N + Mr}, \frac{H - M}{N}\right\}$ ($\min\{h_{\mathcal{CD}}, h_{\mathcal{CD},n}\}$)	$\min\left\{\frac{L}{N + Mr}, \frac{L - M}{N}\right\}$ ($\min\{l_{\mathcal{CD}}, l_{\mathcal{CD},n}\}$)

Note. Decisions: $\mathcal{SD}$ denotes straight bonds, $\mathcal{E}$ denotes equity, and $\mathcal{CD}$ denotes convertible bonds. C denotes "call" convertibles, and N denotes "not call" convertibles. Payoffs: N is the number of old shares; M is the face value and number of convertible bonds issued, and Q is the number of new shares issued. r is the conversion ratio of a single convertible bond. I is the cost of the investment opportunity, H denotes a high payoff, and L denotes a low payoff.

$P(H \mid \Phi_t)$, where Φ_t is the manager's information set at time t. Similarly, the conditional probability that $\tilde{\phi}_1 = G_1$ given ϕ_0 is denoted $P(G_1 \mid \phi_0)$; etc.

Prior unconditional probabilities follows standard notation and are denoted $P(\Phi)$ where Φ is some information set. For example, the probability that $\tilde{\phi}_0 = B_0$, $\tilde{\phi}_1 = G_1$, $\tilde{\phi}_2 = G_2$, and $\tilde{\delta} = H$ is written $P(B_0 G_1 G_2 H)$.

ASSUMPTION 1. ϕ_0 is uninformative given ϕ_1; that is, $P(\phi_2 \mid \phi_0 \phi_1) = P(\phi_2 \mid \phi_1)$ and $P(\delta \mid \phi_0 \phi_1 \phi_2) = P(\delta \mid \phi_1 \phi_2)$ (as long as these probabilities are well defined).[9]

The effect of this is to make an equilibrium in the conversion game independent of the date 0 information of the manager. This is mathematically convenient, but is not necessary for the results of the model.

For G_0, B_0, and A_0 to contain different information, none of $P(G_1 \mid G_0)$,

[9] If, for example, $P(G_1 \mid G_0) = 1$, then $P(\phi_2 \mid G_0 B_1)$ is not well defined.

$P(G_1 \mid B_0)$, and $P(G_1 \mid A_0)$ can be equal. As a convenience of language and without loss of generality, the following assumption is made.

ASSUMPTION 2. $P(G_1 \mid G_0) > P(G_1 \mid B_0) > P(G_1 \mid A_0)$; i.e., the conditional probability of good information at date 1 is higher given good information at date 0 than it is given bad information at date 0 which, in turn, is higher than given awful information at date 0.

Restrictions on the other probabilities are not made at this time. I discuss in Sections 3 and 4 that the separating equilibrium imposes further restrictions; that is, it does not exist for an arbitrary stochastic process of the manager's proprietary information. This allows empirical implications to be derived.

The Manager's Strategies and Investors' Beliefs

At date 0, the manager's strategy is a conditional probability distribution over financing by straight debt, convertible debt, or equity. That is, the strategy specifies the probability, σ_0, of choosing $d_0 \in \{\mathcal{SD}, \mathcal{CD}, \mathcal{E}\}$ given ϕ_0.

For $t = 1$, the strategy specifies the probability, σ_1, of choosing $d_1 \in \{C, N, NoAction\}$ given ϕ_0, ϕ_1, d_0.

For $t = 2$, the strategy specifies the probability, σ_2, of choosing $d_2 \in \{C, N, NoAction\}$ given ϕ_0, ϕ_1, ϕ_2, d_0, d_1.

The manager's optimal strategy depends on the market value of the shares. These depend in turn on the strategy that investors believe that the manager follows. Beliefs are denoted by $P(\Phi \mid D)$ and are the probabilities of any set Φ of the manager's private information as assessed by investors given observed manager decisions, D. For example, suppose convertibles have been issued, but not called in period 1. The probability as assessed by investors that $\tilde{\phi}_0 = B_0$ and $\tilde{\phi}_1 = G_1$ is $P(B_0 G_1 \mid \mathcal{CD}N)$.

The market value of a share at time t is:

$$S_2(\mathcal{CD}d_1d_2) = \sum_{\delta} P(\delta \mid \mathcal{CD}d_1d_2) S_3(\mathcal{CD}d_1d_2\delta) \tag{1a}$$

$$S_1(\mathcal{CD}d_1) = \sum_{\phi_0\phi_1\phi_2 d_2} S_2(\mathcal{CD}d_1d_2)\sigma_2(d_2 \mid \phi_0\phi_1\phi_2\mathcal{CD}d_1)P(\phi_0\phi_1\phi_2 \mid \mathcal{CD}d_1) \tag{1b}$$

$$S_0(\mathcal{CD}) = \sum_{\phi_0\phi_1 d_1} S_1(\mathcal{CD}d_1)\sigma_1(d_1 \mid \phi_0\phi_1\mathcal{CD})P(\phi_0\phi_1 \mid \mathcal{CD}) \tag{1c}$$

$$S_t(\mathcal{SD}) = \sum_{\delta} P(\delta \mid \mathcal{SD}) S_3(\mathcal{SD}\delta), \qquad t = 0, 1, 2 \tag{2}$$

$$S_t(\mathcal{E}) = \sum_{\delta} P(\delta \mid \mathcal{E}) S_3(\mathcal{E}\delta), \qquad t = 0, 1, 2. \tag{3}$$

Notice that if the manager chooses to issue straight debt or equity, the price of a share of stock remains constant until period 3 since no further release of information occurs.

3. SIGNALLING WITH CALLABLE CONVERTIBLE DEBT

3.1. *Separating Equilibrium*

In this section, I discuss the separating equilibrium in which the manager issues straight bonds if he has good information, convertible bonds if he has bad information, and equity if he has awful information. Moreover, if the manager issues convertible bonds, the equilibrium coincides with Harris and Raviv's separating equilibrium in the conversion game; the manager calls if and only if he receives more bad information in the future.

THEOREM 1.[10] *There are values of the exogenous parameters $w(\cdot)$, H, L, N, I, and the prior probabilities such that there exists an equilibrium in which the manager issues straight debt if $\tilde{\phi}_0 = G_0$, convertible debt if $\tilde{\phi}_0 = B_0$, and equity if $\tilde{\phi}_0 = A_0$. Moreover, if convertible debt is issued, the manager calls in period 1 if and only if $\tilde{\phi}_1 = B_1$ and in period 2 if and only if $\tilde{\phi}_2 = B_2$.*

The signalling is driven by the cost the manager suffers if straight or convertible debt is outstanding at date 3 and a low payoff occurs. As an illustration, suppose that $w(\cdot) = \log(\cdot)$. If the face value of the debt is close to L, the date 3 stock price will be close to zero and the manager's utility exceedingly small.[11] The manager can avoid this only by issuing equity or by issuing callable convertibles and forcing conversion prior to maturity. By not taking the safe option, the manager signals good information. The "bad" firm distinguishes itself from the "awful" firm by issuing convertible bonds, thus showing its willingness to bear the costs of signalling via call policy and the possibility of incurring a low payoff at date 3 with debt outstanding.

Existence of the separating equilibrium requires that convertible bondholders will not convert voluntarily in some states of the world. Specifically, it is necessary that $rl_{\mathscr{CL}} < 1$; the conversion value of a single convertible in the case of a low firm value at maturity must be smaller than its face value. If this were not the case, convertible bondholders would convert voluntarily in period 3—if conversion were not already forced—whether the payoff

[10] The proof of Theorem 1, like all proofs not supplied in the text, is given in the Appendix.

[11] The cost here is very much like a bankruptcy cost. However, if the manager's utility function is $w(y) = \log(y - \Theta)$, where $\Theta > 0$ is a constant, then the manager's utility is arbitrarily small if the stock price is close to Θ. The interpretation is that there is a large cost imposed upon the manager if the firm's share price drops to some threshold. The source of this sort of cost may be related to the manager losing his job, status, or power if the share price declines "too much."

were high or low. The payoff to stockholders in period 3, therefore, would be independent of the manager's call decisions, implying that these could not serve as credible signals of information. With voluntary conversion guaranteed, there would also be no real distinction between convertible debt and equity, implying that these two instruments could not signal different information. Thus, $rl_{\mathcal{CD}} < 1$.

Under the empirically plausible assumption that a convertible bond should sell at a premium over its straight debt value, the separating equilibrium also requires that there are some states of the world in which convertible bondholders *will* convert voluntarily. Thus, it is necessary that $rh_{\mathcal{CD}} > 1$; the conversion value of a single convertible in the case of a high firm value at maturity must be larger than its face value. If this did not hold, a convertible bond would sell at a discount to its straight debt value since there is a positive equilibrium probability of forced conversion prior to maturity. Thus the result that $rh_{\mathcal{CD}} > 1$ is directly linked to the assumption that $M < I$; the total face value of the convertible bonds is smaller than their total value (which equals the cost of investment) at date 0. To see this formally, in the separating equilibrium, the value of the convertibles is given by

$$M[(P_{H|\phi_0} - P_{G_1G_2H|\phi_0})rh_{\mathcal{CD}} + P_{G_1G_2H|\phi_0}\max\{rh_{\mathcal{CD}}, 1\}]$$
$$+ M[(P_{L|\phi_0} - P_{G_1G_2L|\phi_0})rl_{\mathcal{CD}} + P_{G_1G_2L|\phi_0}],$$

which should equal I. But since $rl_{\mathcal{CD}} < 1$ and $M < I$, this is impossible unless $rh_{\mathcal{CD}} > 1$.

In practice, there is an inverse relation between the straight debt value and the dilution factor of a convertible issue; that is, the face value (or coupons) can usually be lowered by increasing the conversion ratio. In the separating equilibrium, this inverse relation also exists provided that the probability of a high payoff given good information at date 2 is less than 1. If, on the contrary, $P(H\,|\,G_2) = 1$, conversion is certain; for at date 2, if the convertibles have not yet been called and the manager's information is G_2, he can be certain of voluntary conversion since $rh_{\mathcal{CD}} > 1$. On the other hand, if the manager's information is B_2, he will force conversion. When conversion is certain, the face value of the convertibles will not affect their value. Essentially, the convertible bonds would be a form of delayed equity. Nevertheless, as shown in the proof of Theorem 1, the dilution with convertible bonds will be lower than that with an issue of equity, since convertible bonds signal more favorable information. This result is similar to Stein's (1992). However, my model also accommodates the case where conversion is uncertain, and there is a tradeoff between the conversion ratio and face value of convertible bonds.

Existence of the separating equilibrium also calls for restrictions on the prior probabilities. Harris and Raviv (1985) show that the separating equilibrium in the conversion game following an issue of callable convertible debt requires that

$$P(H \mid G_1G_2) > P(H \mid G_1B_2) \quad \text{and} \quad P(H \mid G_1) > P(H \mid B_1). \tag{4}$$

Since in equilibrium a call of convertible debt at time t is associated with the private information B_t, Eq. (4) implies that a call will cause a drop in the stock price. This is consistent with Mikkelson's (1981) evidence.

Combining (4) with Assumption 2 implies that $P(H \mid G_0) > P(H \mid B_0) > P(H \mid A_0)$. Thus the separating equilibrium of Theorem 1 is consistent with the observed evidence that the average abnormal return associated with the announcement of a straight debt issue is greater than that associated with the announcement of a convertible debt issue which, in turn, is greater than the average abnormal return associated with an equity issue.

An important question is whether there exist other equilibria. For now I will only consider other separating equilibria, leaving the issue of pooling equilibria for a later section.

THEOREM 2. *If the values of the exogenous parameters and the prior probabilities are such that the separating equilibrium described in Theorem 1 exists, it is also the unique separating equilibrium consistent with the separating equilibrium in the conversion game.*

I will prove the theorem with two lemmas, the first addressing signalling with equity and debt, and the last focusing on straight versus convertible debt.

As discussed in the Introduction, issuing equity provides a better insurance against adverse stock price movements than issuing debt. This is the key to the following result.

LEMMA 1. *In any separating equilibrium consistent with the separating equilibrium in the conversion game, both convertible and straight debt signal more favorable information than equity.*

Proof. If there were no information asymmetry at the time of financing, the manager would prefer equity to either straight or convertible debt. This follows by observing that while the expected stock price at date 3 is the same regardless of the issued security, the variance of the date 3 stock price would be the smallest following a date 0 issue of equity. Additionally, if callable convertible debt were issued and there were information asymmetry at dates 1 and 2, the stock prices at these dates would have a larger variance than if equity had been issued because of signalling through call policy. The preferernce for equity follows from the concavity of the manager's utility function.

Return now to the case where the manager has private information at date 0: if straight debt were associated with less favorable information than equity, the manager could improve his welfare by deviating from such a proposed equilibrium by issuing equity when he supposedly should issue debt. His gain would be twofold: (i) an overvalued stock price at dates 0, 1, and 2; and (ii) and a lower variance of date 3 stock price. Similarly, if convertible debt were associated with less favorable information than equity, the manager could improve his welfare by issuing equity when he supposedly should issue convertible debt. His gain would still be twofold: (i) an overvalued initial stock price, (ii) a lower variance of date 1, 2, and 3 stock prices. It follows that in any separating equilibrium, equity must be associated with less favorable news than either of the two forms of debt. ∎

Concavity of the manager's welfare function plays on obviously important role in the proof of this lemma since it causes the manager's preferred security to be equity. The tradeoff between adverse selection and insurance that is at the heart of Lemma 1 would also suggest that convertible debt would be the manager's preferred debt security. If so, convertible debt could not signal more favorable information than straight debt. This is addressed in the next lemma.

LEMMA 2. *If the values of the exogenous parameters and prior probabilities are such that the separating equilibrium described in Theorem* 1 *exists, then there is no equilibrium consistent with the separating equilibrium in the conversion game in which convertible debt signals more favorable information than straight debt.*

Proof. The proof will take as given that the separating equilibrium described in Theorem 1 exists. This implies that the manager's preferred debt security given B_0 must be convertible debt. To see this, suppose on the contrary that the manager's preferred debt security is straight debt. In this case, the manager would prefer issuing straight debt upon observing B_0 rather than convertible debt as prescribed by the equilibrium. The manager's gains would be twofold: (i) an initially overvalued stock price (since investors would interpret an issue of straight debt as the manager's proprietary information being G_0), and (ii) improved insurance against adverse stock prices (since, by hypothesis, straight debt is the preferred debt security).

The above establishes that if B_0 is common knowledge, the manager's preferred debt security is convertible debt. This implies that convertible debt cannot signal more favorable information than straight debt since if investors inferred more favorable information upon observing a convertible debt issue than a straight debt issue, a manager knowing B_0 could issue his preferred type of debt at an overvalued price. ■

The intuition of the lemma relates to the adverse selection versus insurance tradeoff discussed earlier. In a separating equilibrium, a manager knowing G_0 issues straight debt despite the resulting cost of being poorly insured against adverse stock prices in the future. The benefit of straight debt is that it has a smaller adverse selection problem.

Lemmas 1 and 2 establish Theorem 2. Under symmetric information at date 0, there is a pecking order of the choice of financing: the manager prefers issuing equity to debt, and he prefers convertible debt to straight debt. When the manager has private information at date 0, a separating equilibrium reverses this pecking order due to adverse selection—equity signals less favorable information than convertible debt which, in turn, signals less favorable information than straight debt.[12]

Remark. Signalling in the conversion game can only take place if the payoff to unconverted convertible bonds at maturity is sufficiently high to have a significant negative impact on share price if the firm is in a "slump." For signalling with straight debt to take place at the financing stage, the same must be true for the face value of the straight debt.

Separation between convertible debt and equity is possible only if awful information and bad information are sufficiently different. For a given $P(G_1 | B_0)$, separation between convertible debt and equity imposes restrictions on $P(G_1 | A_0)$—it must be strictly less than $P(G_1 | B_0)$. However, just any number less than $P(G_1 | B_0)$ will not do. If $P(G_1 | A_0)$ is sufficiently close to $P(G_1 | B_0)$, the separating equilibrium does not exist because the manager prefers to issue equity rather than convertibles (assuming that the separating equilibrium obtains in the conversion game). $P(G_1 | A_0)$ is also bounded below. Given the beliefs of the separating equilibrium, if $P(H | A_0)$ were smaller than $P(H | B_1)$ and the manager knew A_0, he would do better by issuing convertibles and calling them in period 1 than by issuing equity

[12] If the values of the exogenous parameters do not support the separating equilibrium, the manager's preferred debt security when B_0 is common knowledge could actually be straight debt since under the separating equilibrium in the conversion game, the variability of the stock price at dates 1 and 2 will be larger following an issue of convertible debt than following an issue of straight debt. This increased stock price variability at the intermediate dates could offset the intrinsic advantage of convertible debt's lower variability at maturity (date 3). Thus, when the manager has proprietary information at date 0, there may exist a separating equilibrium in which convertible debt signals more favorable information than straight debt, depending on the values of the exogeneous parameters and prior probabilities. However, it can be shown that if $P(H | G_0) = 1$, straight debt must signal more favorable information than convertible debt in any separating equilibrium. It will be shown in a later section that this condition also relates to the uniqueness of the separating equilibrium of Theorem 1 when equilibrium refinements are applied.

(see Proposition 3). Hence, $P(H|A_0) > P(H|B_1)$ which implies that $P(G_1|A_0)$ must be strictly greater than 0.[13] Thus for separation to obtain at the financing stage, it is necessary that $P(G_1|A_0) \in (a, b)$ with $a > 0$ and $b < P(G_1|B_0)$, where a and b depend on the values of the other probabilities and parameters.

3.2. *Callable versus Noncallable Convertible Debt*

The advantage of callable, as opposed to noncallable, convertible debt is that it increases the likelihood of conversion into equity, since the call feature allows a firm to force conversion in some states of the world where conversion otherwise would not take place. Thus callable convertible debt has better insurance properties against adverse stock price movements. One can think of the call feature as lowering the expected costs of financial distress. The disadvantage of callable convertible debt is that in an equilibrium where call policy signals a firm's quality, the stock price is more volatile than it would be otherwise. This imposes a cost on managers and other stakeholders if they have a preference for smoother stock prices, as they do here. Managers would favor callable convertible bonds to the noncallable variant if the costs of financial distress were sufficiently high.

In the separating equilibrium, if $P(H|G_2) = 1$, convertible debt is essentially delayed equity since conversion is guaranteed. The expected costs of financial distress incurred by issuing callable convertible debt would thus be nil. Strictly speaking, there is no bankruptcy in the model since the payoff in the low state is assumed larger than the face value of convertible debt, $L > M$. However, a low payoff involves costs which we could interpret as costs of financial distress. This was illustrated above by considering the example where the manager's utility function is represented by $w(S_t) = \log S_t$. The combination of a low liquidating dividend and nonconverted convertibles would give the manager a utility of

$$\log\left(\frac{L - M}{N}\right)$$

as opposed to

$$\log\left(\frac{L}{N + Mr}\right)$$

[13] To see this: $P(H|A_0) > P(H|B_1) \Leftrightarrow P(G_1|A_0)P(H|G_1) + P(B_1|A_0)P(H|B_1) > P(H|B_1) \Leftrightarrow P(G_1|A_0)P(H|G_1) > P(G_1|A_0)P(H|B_1) \Leftrightarrow P(G_1|A_0) > 0$, since $P(H|G_1) > P(H|B_1)$.

if the convertibles had been converted. The former event can thus be seen to lower the manager's utility, the more so the closer M is to L. While this "financial distress" event would not occur under callable convertible debt in the case considered, it could occur under noncallable convertible debt.

A firm which has issued noncallable convertible debt could at a later date attempt to protect itself against financial distress by an equity-financed repurchase. In the context of the current model, suppose that at date 2, the manager observes B_2. An equity-financed repurchase of noncallable convertibles would not have the same favorable effect as forced conversion of callable convertibles. This is because the repurchase price would be calculated conditional on B_2 (in a separating equilibrium) while the implicit repurchase price in a forced conversion would be calculated conditional on B_0. Given that the manager's information becomes more precise over time, the dilution of existing equity would thus be higher in the case of the repurchasing of the noncallable convertible debt.[14] For example, if $P(L \mid B_2) = 1$, the stock price after a repurchase of convertible debt at date 2 would be no different than it would be at date 3 if no repurchase took place.

The discussion illustrates the superior insurance properties of callable convertible debt compared to noncallable convertible debt. The expected costs of financial distress are higher with noncallable than with callable convertibles, although these costs are understated for callable convertibles in the current model since the call price is assumed equal to zero. Callable convertible debt would thus be preferred to noncallable convertible debt as a signal of intermediate quality. This provides a rationale for callable convertibles in Harris and Raviv's (1985) conversion game as well as for the fact that the vast majority of convertible debt issues in the United States have a call feature.

3.3. *Forcing Conversion*

Conversion can be forced with certainty in my model since the call price is assumed to be equal to zero. The model could be adapted to include a higher call price and the possibility of a firm not being able to force conversion by allowing for assets in place whose value would vary over time. The analysis would then explicitly address the issue focused on by Stein (1992): the possibility of not being able to force conversion as a deterrent to issuing convertible bonds. However, in this extended model, when the conversion value is smaller than the call price, conversion into equity may still be achieved by simultaneously calling and offering to swap the convertibles for shares having a larger value than the call price. Essentially, the firm can

[14] It is a necessary condition of the separating equilibrium of Theorem 1 that the manager's information becomes more precise over time in the sense that $P(H \mid G_t) > P(H \mid G_\tau)$ and $P(L \mid B_t) > P(L \mid B_\tau)$ for $t > \tau$. See Section 4.

increase the dilution factor at its discretion (assuming that the contractually specified dilution factor is not 100%). Intuition based on the current model would suggest that this kind of convertible repurchase would only be performed by firms with relatively pessimistic forecasts. This contrasts with Stein's (1992) model where a convertible repurchase by a low-quality firm is ruled out, since, by assumption, the firm will go bankrupt with certainty once the conversion value is below the call price.

Whether poorly performing firms that do not repurchase their convertibles choose not do so (as suggested here) or do not have sufficient assets to do so (as in Stein's model) is an empirical question. Brennan and Her (1993) report that the average dilution factor in their sample of convertible bonds is 16.3%, suggesting that there is ample scope for a convertible to equity swap. Additionally, among COMPUSTAT firms with convertibles outstanding in July 1994, the average leverage ratio (debt to total assets) is 42%. For these firms, convertible debt amounted to 43% of all debt and 16% of assets.[15]

3.4. *Call Protection Periods*

Convertible bonds are often call protected for some time after issuance.[16] This may reduce the probability of conversion. Perhaps the most obvious reason is that the conversion value may rise above the call price and then fall below it again while the convertibles are call protected. However, this is not a concern in the current analysis since forced conversion is assumed to be implementable at both date 1 and date 2. Call protection at date 1 may nevertheless reduce the probability of conversion. For example, without call protection, if the manager's proprietary information is B_1 at date 1, he will force conversion (in the separating equilibrium). On the other hand, if the convertibles are call protected, the manager would have to wait until date 2. If he then receives the information G_2, he will choose not to call (in a separating equilibrium). But if the subsequent value of the firm at the maturity of the convertibles turns out to be "low," investors will not convert voluntarily (as shown earlier). As a result, call protection at date 1 reduces the probability of conversion by $P(B_1G_2L)$. Thus, signalling causes the call protection features to worsen the insurance properties of the convertible debt.

[15] The July 1994 COMPUSTAT tapes list 712 firms with convertibles outstanding. I have excluded firms with noncallable convertible debt as reported by *Moody's Bond Record* (July 1994). The total sample is then reduced to 697. For all but 9 out of 491 convertibles listed by *Moody's*, the call price is less than 110 per 100 of face value. I thank Carolina Minio Paulino for doing the leverage calculations.

[16] Some convertibles have a "soft call" feature whereby the stock price must exceed the conversion price by a certain percentage for the firm to be able to call.

Call protection at date 1 imposes an additional cost because bad information at date 2 implies a higher likelihood of a low payoff than bad information at date 1. Thus, a call at date 2 results in a lower stock price than a call at date 1.

Call protection at date 2 is also possible. Whether call protection at date 1 or at date 2 is preferable to the manager may depend on the information process. An advantage with date 2 call protection is that forced conversion would come at an earlier date, and therefore have a less severe impact on the stock price. On the other hand, signalling implies that the probability of conversion is reduced by $P(G_1B_2L)$ relative to the fully callable convertible. In practice, one would also have to look at the effect of the call protection period on the probability of being able to force conversion.

A mitigating factor of call protection is that the lower probability of conversion leads to a lower dilution for a given face value of the bonds. But if the costs of financial distress were sufficiently high, managers would prefer no call protection.

It is somewhat puzzling why many convertibles are issued with call protection features. Perhaps the length of the call protection period serves as a signal; the longer it is, the more favorable the signal. This could be incorporated into the current model by allowing for more values of the manager's initial information. A test of this hypothesis could compare the announcement abnormal returns to the relative length of the call protection period. Noncallable convertible debt would be expected to exhibit the least negative impact.

4. PREDICTIONS OF THE SEPARATING EQUILIBRIUM

This section discusses the empirical implications of the separating equilibrium. The separating equilibrium is consistent with key elements of the empirical evidence regarding external financing and calls that force conversion of convertible debt into equity. Additionally, the section discusses the prediction that issuing callable convertibles and later forcing conversion has a greater adverse effect on the stock price than issuing equity at the outset. The initial benefit of issuing callable convertibles as opposed to equity is preserved only if conversion into equity is voluntary. Since the current model encompasses Harris and Raviv's (1985), all predictions of that model remain.

The first prediction relates the reaction of a firm's stock price to the firm's choice of financing. This is consistent with the empirical evidence supplied by Smith (1986).

PROPOSITION 1. *In the separating equilibrium, the stock price is highest after a straight debt issue and lowest after an equity issue. That is,* $S_0(\mathcal{SD}) > S_0(\mathcal{CD}) > S_0(\mathcal{E})$.

The reason for this ordering is that in equilibrium straight debt signals more favorable information than convertible debt which, in turn, signals more favorable information than equity, inducing a better stock price reaction. This leads to the next result.

PROPOSITION 2. *In the separating equilibrium, the cash flow to the firm in period 3 conditional on straight debt being issued (first order) stochastically dominates the cash flow to the firm in period 3 conditional on convertible debt being issued which, in turn, (first order) stochastically dominates the cash flow to the firm in period 3 conditional on equity being issued.*

Since the value of convertible debt is bounded below by its conversion value, if there is no information asymmetry at the time of financing (or any other market imperfections), the number of shares into which an issue of convertible debt can be converted must be smaller than the number of shares required from an equity issue raising the same amount of capital. This is reinforced in the separating equilibrium since convertible debt is associated with more favorable news than equity (Proposition 1; see also Lemma 1). Thus:

LEMMA 3. *In the separating equilibrium, $Mr < Q$. That is, the number of shares into which an issue of convertible debt can be converted is smaller than the number of shares required in an issue of equity.*

In conjunction with Proposition 1, Lemma 3 implies that *calling* convertible debt signals worse information than issuing equity. For if not, the manager would always do better by issuing convertibles and later forcing conversion than by issuing equity at the outset; the share price would be higher every period with the former strategy because it would be interpreted as the manager having more favorable information and because of the smaller dilution. This would contradict the existence of the separating equilibrium. Thus:

PROPOSITION 3. *In the separating equilibrium, the stock price is larger after an issue of equity than it is after a call of convertible debt. That is, $S_0(\mathscr{E}) > S_1(\mathscr{C}\mathscr{D}C)$ and $S_0(\mathscr{E}) > S_2(\mathscr{C}\mathscr{D}NC)$.*

The proposition says that issuing equity outright has a less adverse effect on the stock price than issuing convertibles and later forcing conversion. This has to do with the temporal resolution of uncertainty. In equilibrium, issuing equity and calling convertibles are both signals of poor information. However, since calling convertibles would come at a relatively later date ($t = 1, 2$) when management's information would be more prrecise, investors interpret it more negatively than they would an outright issuing of equity (at $t = 0$). The proposition relates to the event-study evidence mentioned earlier that the combined AAR from issuing convertible debt and later forcing conversion is more negative than the AAR from issuing equity.

Propositions 1 and 3 indicate that issuing convertible debt as delayed equity is *ex post* efficient only if conversion is voluntary. Put differently, a strategy of issuing convertibles with the intention of forcing conversion leads to a more adverse price impact than issuing equity outright. This has important implications for the view that callable convertible debt is delayed equity. In my model, callable convertible debt is not equivalent to delayed equity; rather, it can be viewed as delaying the decision to be in debt or equity, with the hope that investors voluntarily choose equity. The call feature offers insurance against adverse stock prices if in the future voluntary conversion looks unlikely. In a sense, callable convertible debt postpones the adverse selection problem of equity. In the model, voluntary conversion takes place at $t = 3$ provided a high payoff obtains. This may be interpreted as a date at which dividends are greatly increased.

As observed in the proof of Lemma 1, future stock prices are more variable subsequent to an issuing of convertible debt as compared to an issuing of equity, due to the private information released by the manager through his call policy. Letting $R_t(\mathcal{CD}) = S_t(\mathcal{CD})/S_{t-1}(\mathcal{CD})$, $R_t(\mathcal{SD}) = S_t(\mathcal{SD})/S_{t-1}(\mathcal{SD})$, and $R_t(\mathcal{E}) = S_t(\mathcal{E})/S_{t-1}(\mathcal{E})$:

PROPOSITION 4. *In the separating equilibrium, the variance of stock rates of return in periods* 1 *and* 2 *following an issue of convertibles is larger than following an issue of equity or straight debt. That is, for* $t = 1, 2$ $\mathrm{Var}[R_t(\mathcal{CD})] > \mathrm{Var}[R_t(\mathcal{E})]$ *and* $\mathrm{Var}[R_t(\mathcal{CD})] > \mathrm{Var}[R_t(\mathcal{SD})]$.

It is difficult to say something about the comparison of these variances in period 3 without restricting the prior probabilities. Intuitively, it may seem that the period 3 rate of return should be lower after an issue of convertibles than after an issue of equity since the manager's call decisions "speed up" the resolution of uncertainty. However, nothing precludes the variance of the project payoff ($\tilde{\delta}$) to be smaller conditional on equity being issued than conditional on convertible debt being issued; i.e., the initial uncertainty may be larger with convertibles than with equity (in the separating equilibrium). But if the call decisions resolve uncertainty fast enough, the variance of the return in period 3 will be lower with convertibles than with new equity. For example, if $P(H \mid G_2) = 1$ and $P(L \mid B_2) = 1$, the variance of the period 3 rate of return following an issue of convertible bonds will be 0.

These propositions assert that the separating equilibrium of Theorem 1 requires that a good signal at a later date is a more precise indicator of a high payoff than a good signal at an earlier date. Likewise, a bad signal at a later date is a more precise indicator of a low payoff than a bad signal at an earlier date. In other words, the separating equilibrium exists only when the information process has the intuitively appealing property that information becomes more precise over time.

5. ROBUSTNESS OF THE SEPARATING EQUILIBRIUM

This section examines the other equilibria of the model when (i) the values of the exogenous parameters are such that the separating equilibrium exists, and (ii) the information process has the Markov property; that is, $P(\phi_2 | \phi_0\phi_1) = P(\phi_2 | \phi_1)$ and $P(\delta | \phi_0\phi_1\phi_2) = P(\delta | \phi_2)$. The next two theorems give conditions under which the separating equilibrium of Theorem 1 is unique, where pooling equilibria are eliminated by Cho and Kreps (1987) *intuitive criterion* (see also Kreps, 1985) and Banks and Sobel's (1987) *divinity* refinement.

THEOREM 3 (Uniqueness in Conversion Game). *If $P(G_2 | G_1) = P(H | G_2) = 1$ and the values of the other exogenous parameters and probabilities are such that the separating equilibrium in the conversion game exists, the separating equilibrium is unique up to an application of the intuitive criterion.*

This shows that for the separating equilibrium to be unique, it is sufficient that once the manager receives good information, he is almost certain that the project will return a high payoff. Next, I show that one additional probability restriction is sufficient to eliminate all nonseparating equilibria in pure strategies in the full game.

THEOREM 4 (Uniqueness in Full Game). *Suppose $P(G_t | G_{t-1}) = 1$ for $t = 1, 2$ and $P(H | G_2) = 1$. Let the values of the other exogenous parameters and probabilities be given such that the separating equilibrium of Theorem 1 exists. There is an $x \in (0, 1)$ such that if the unconditional probability of awful information at date 0, $P(A_0)$ is greater than x, then the separating equilibrium is unique among pure strategy equilibria up to an application of the intuitive criterion and divinity.*

The proof is sketched below.

First, when $P(G_t | G_{t-1}) = P(H | G_2) = 1$ and the values of the other parameters and probabilities are chosen such that the separating equilibrium exists, then when the unconditional probability of awful information at date 0 is sufficiently large, any equilibrium will have the manager issuing equity upon receiving awful information. This assertion derives from equity being the manager's preferred security; when at date 0 his information is A_0, the manager issues equity unless there are large gains from issuing another security and being pooled with higher types. But when the unconditional probability of A_0 is sufficiently large, these pooling gains become trivially small.

TABLE II
EQUILIBRIUM STRATEGIES IN THE FULL GAME

	Strategies		
	A_0	B_0	G_0
Equilibrium I[a]	$\mathscr{E}$	$\mathscr{E}$	$\mathscr{E}$
Equilibrium II[a]	$\mathscr{E}$	$\mathscr{E}$	$\mathscr{SD}$
Equilibrium III[a]	$\mathscr{E}$	$\mathscr{SD}$	$\mathscr{SD}$
Equilibrium IV[b]	$\mathscr{E}$	$\mathscr{CD}$	$\mathscr{CD}$
Separating equilibrium	$\mathscr{E}$	$\mathscr{CD}$	$\mathscr{SD}$

Note. Attention is limited to pure strategies where equity is issued if $\tilde{\phi}_0 = A_0$. This can be motivated by the fact that equity is the manager's preferred security combined with the assumption that the unconditional probability of A_0 is "large" (see text). It is assumed that the separating equilibrium obtains in the conversion game.

[a] The equilibrium is eliminated by divinity if $P(A_0)$ is sufficiently large that $P(A_0)P(H|A_0) + (1 - P(A_0))P(H|G_0) \leq P(H|B_0)$. (See footnote 18.)

[b] The equilibrium is eliminated by the intuitive criterion if $P(G_t|G_{t-1}) = 1$ and $P(H|G_2) = 1$.

If equity is issued when the manager's initial information is awful, there are only five possible pure strategy equilibria. These are listed in Table II (a proof that other combinations cannot be equilibria is available upon request).

Consider first Equilibrium IV. In this equilibrium, the manager issues equity if he has awful information and convertible debt otherwise. Since the separating equilibrium exists, no matter how an issue of straight debt is interpreted, the manager prefers not to issue straight debt if $\tilde{\phi}_0 = A_0$ or B_0. However, if $\tilde{\phi}_0 = G_0$ and if $P(G_1|G_0) = 1$ etc., the manager would prefer to issue straight debt rather than convertible debt as prescribed by the equilibrium, provided that investors interpret an issue of straight debt as $\tilde{\phi}_0 = G_0$. Thus, Equilibrium IV is eliminated by the intuitive criterion.

Eliminating the other equilibria requires more work. As an example, consider Equilibrium III. It is clear that if investors place a zero probability on $\tilde{\phi}_0 = A_0$ upon observing the out-of-equilibrium action of a convertible debt issue, Equilibrium III breaks down—since if $\tilde{\phi}_0 = B_0$, the manager would do better by issuing convertibles rather than straight debt. The key to eliminating Equilibrium III, therefore, is to argue that it is unreasonable to place a positive probability on $\tilde{\phi}_0 = A_0$ if convertibles are issued. However, without further restrictions on the parameters, the intuitive criterion cannot be employed to this end; it is necessary to use stronger tools.[17] Next

[17] For the intuitive criterion to eliminate Equilibrium III, the parameter values must be such that if $\tilde{\phi}_0 = A_0$, the manager does better by issuing equity than by issuing convertibles even if the latter is interpreted by investors as $\tilde{\phi}_0 = G_0$.

I show that Equilibrium III can be eliminated by divinity provided that $P(A_0)$ is sufficiently large.

Banks and Sobel's (1987) divinity refinement is based on the same type of out-of-equilibrium inference behavior as the intuitive criterion. In particular, if in Equilibrium III investors are confronted with a convertible debt issue, they should question the reasonableness of their beliefs according to which convertible debt should never be issued. Beliefs must be revised. Divinity imposes a consistency criterion to the (revised) out-of-equilibrium beliefs as follows: if the beliefs induce the manager to deviate if $\tilde{\phi}_0 = A_0$, the probability assigned by investors that a deviation comes from a manager knowing A_0 must be at least the prior, $P(A_0)$. But if this is sufficiently large, the manager will not deviate after all (again since the separating equilibrium exists).[18] Thus, for sufficiently large $P(A_0)$, any reasonable out-of-equilibrium beliefs must be such that the manager does not deviate if his information is A_0. But then these same beliefs must place a zero probability on $\tilde{\phi}_0 = A_0$ if convertibles are issued. Thus we can conclude that any reasonable out-of-equilibrium beliefs must be such that the manager deviates from the equilibrium if $\tilde{\phi}_0 = B_0$. In other words, Equilibrium III cannot be supported by reasonable out-of-equilibrium beliefs. This is the essence of the Banks and Sobel (1987) divinity refinement: equilibria that cannot be eliminated for infinite iterations of such arguments are called *divine*. Equilibrium III is thus an example of an equilibrium which is not divine. Virtually the same argument establishes that Equilibrium I and Equilibrium II also are not divine.

To summarize, if good information is very likely to be followed by more good information, there is little reason to expect any equilibrium other than that of Theorem 1 (as long as the other parameter values are such that this equilibrium exists).

6. DISCUSSION

6.1. *Dividend and Call Policy*

The benefits of convertible debt as delayed equity are preserved only if voluntary conversion is achieved. In the context of my model, voluntary conversion takes place at the maturity of the convertibles, but this could

[18] Since the parameter values are such that the separating equilibrium of Theorem 1 exists, under A_0 the manager would only have an incentive to deviate from Equilibrium III by issuing convertibles if the out-of-equilibrium beliefs are such that $S_0^{\mathrm{III}}(\mathscr{CD}) > S_0^{\mathrm{SEP}}(\mathscr{CD})$, where $S_0^{\mathrm{III}}(\mathscr{CD})$ denotes the time 0 stock price conditional on the out-of-equilibrium beliefs, and $S_0^{\mathrm{SEP}}(\mathscr{CD})$ denotes the time 0 stock price in the separating equilibrium (where an issue of convertibles is interpreted as $\tilde{\phi}_0 = B_0$). Hence, it is sufficient that the priors satisfy $P(A_0)P(H|A_0) + (1 - P(A_0))P(H|G_0) \leq P(H|B_0)$ because then $S_0^{\mathrm{III}}(\mathscr{CD}) \leq S_0^{\mathrm{SEP}}(\mathscr{CD})$ for any out-of-equilibrium beliefs with $P(A_0|\mathscr{CD}) \geq P(A_0)$.

be interpreted as a date when the firm increases dividends (perhaps for informational reasons), thus stimulating voluntary conversion. Recent empirical contributions by Asquith and Mullins (1991) and Campbell *et al.* (1991) have suggested that the level of dividends in relation to the (aftertax) coupon payments on convertible debt plays an important role in management's decision whether to call and force conversion.

Campbell *et al.* (1991) argue that management would call only when the anticipated dividend to the convertibles if converted would be smaller than the aftertax coupons. They also present evidence that precall earnings growth is typically abnormally large relative to industry norms, and that postcall earnings growth typically falls to the industry norm. This seems to be consistent with the current model, and also that of Harris and Raviv (1985). However, Campbell *et al.* (1991) argue otherwise:

> This evidence in conjunction with the large difference between the conversion value and the adjusted call price is also inconsistent with signaling models in which managers force conversion because they expect that the conversion value will be less than the face value at maturity. (p. 1323)

However, one could view the dividend at date 3 in my model as representing the present value of a perpetual dividend flow. The face value of the convertibles in the model could similarly be viewed as representing the present value of the coupons of a consol. With this interpretation, Campbell *et al.*'s (1991) findings are not at odds with my model—the manager forces conversion prior to date 3 because the anticipated dividends to common stock are not sufficiently large compared to the coupon of the convertibles to induce voluntary conversion. Campbell *et al.*'s (1991) evidence that there are positive abnormal returns prior to a call are also consistent with my model (as well as Harris and Raviv's model, 1985).

While the above discussion takes the anticipated dividends as given, management typically has discretion over dividend payments. By increasing dividends, management can stimulate conversion without recourse to outright forced conversion and thereby avoid the negative stock price reaction typically associated with forced conversion. Where the dividend is internally financed, this may involve a tradeoff with underinvestment (see, e.g., Miller and Rock, 1985).[19] The benefits from paying "high" dividends would be fourfold: (i) increased likelihood of conversion; (ii) a "bird in the hand" effect: if convertible bond holders do not convert, more of the firm's earnings would accrue to current equityholders (retained earnings will eventually be shared with convertible holders when and if they convert); (iii) high

[19] Dividend increases could also be financed by a rights issue. Since most convertible issues have antidilution clauses, a dividend financed by a rights issue would typically not have a conversion stimulating effect.

dividends would presumably be a positive signal; and, related to the last point, (iv) if the convertibles are voluntarily converted, the negative signalling effect associated with a conversion forcing call would presumably be avoided. The cost of paying high dividends to stimulate conversion might be underinvestment and, in some jurisdictions, double taxation of corporate earnings.

One could endogenize dividend policy and study its interaction with call policy in my model by including earnings and investment opportunities at the intermediate dates, with the manager being better informed than investors. From Miller and Rock (1985), we know that in this kind of model there are equilibria in which there is a tradeoff between net dividends and investments. With convertible debt in the capital structure, net dividends (defined so as to include coupon payments) could be reduced by calling the convertibles when the call price is below conversion value, provided that the dividends paid to the new shares would be lower than the coupons paid to the convertibles if not converted. In a Miller and Rock type of separating equilibrium, this would be interpreted by the market as bad news and would have a negative impact on the firm's share price. While net dividends could also be reduced by cutting the dividend to common stock, forcing conversion of convertible bonds has the additional effect of simultaneously extinguishing the put element of the convertible. This model has isolated the implications of this second effect on signalling through convertible debt call policy and also on the initial financing decision.

My model relates the signalling effect of call policy to delays in forced conversion. Asquith and Mullins (1991) cast doubt on the empirical evidence on call delays by demonstrating that a large proportion of firms with outstanding callable convertible debt have a cash flow advantage from not calling. However, this cash flow advantage could easily disappear if dividends were reduced. The puzzle seems to be why firms do not simultaneously reduce dividends and call outstanding convertibles when the conversion value is greater than the call price. The answer might be that both of these actions are interpreted as bad news by the market. Campbell *et al.* (1991) find that 2-day abnormal returns around the announcement of conversion forcing calls are significantly negative only in the case in which the dividend to be received on conversion is smaller than the coupon on the bond. When the dividend is larger than the coupon, a conversion forcing call would, in effect, increase net dividends and as such the call should be good news. But Campbell *et al.* (1991) find negative but statistically insignificant abnormal returns upon a conversion forcing call that increases net dividends. This suggests that while a call may have net dividend implications, the signalling effect focused on in this paper is also important empirically.

6.2. *Concluding Remarks*

I have developed a signalling model of convertible debt call policy and external financing. The model is consistent with most stylized facts regarding financing and call announcement effects and call delays. I address the role of callable convertible debt as delayed equity and conclude that issuing equity via first issuing callable convertible bonds and later forcing conversion has a larger adverse price impact than issuing equity outright. The benefits of convertible debt as delayed equity are realized only if voluntary conversion is achieved. This contrasts with Stein (1992) who argues that it is the possibility of forcing conversion that makes convertible debt attractive as a form of delayed equity. The empirical evidence surveyed in Section 1 supports the interpretation offered here. In the context of my model, voluntary conversion takes place at the maturity of the convertibles, but it could be interpreted as a date when the firm increases dividends (perhaps for informational reasons), thus inducing voluntary conversion.

The basic tradeoff in the model is between insurance (against costly bankruptcy) and adverse selection. This tradeoff drives the separating equilibrium in which straight debt is associated with better news than callable convertible debt which, in turn, is associated with better news than equity. For example, callable convertible debt offers worse insurance but better adverse selection properties than equity. The same intuition would suggest that noncallable convertible debt should be associated with worse news than callable convertible debt. This conjecture can be tested empirically using the standard event study methodology.

APPENDIX

Proof of Theorem 1. First, I will prove that the set of exogenous parameter values for which the equilibrium holds is nonempty by presenting a numerical example in which the stated separating equilibrium exists. Second, I will demonstrate that given values for some of the parameters and probabilities, one can always find values for the remaining parameters and probabilities such that the separating equilibrium exists. Finally, I will argue that the equilibrium derived here is sequential.

The numerical example as well as the longer analytical proof will assume the following information process:

$$P(G_t \mid G_{t-1}) = P(H \mid G_2) = 1 \quad \text{and} \quad P(H \mid B_2) = 0, \tag{A.1a}$$

$$0 < P(G_t \mid B_{t-1}) \equiv P_{G|B} < 1. \tag{A.1b}$$

Let also each of $P(A_0)$, $P(B_0)$, and $P(G_0)$ be strictly greater than zero.

Proof by numerical example. Let M and the exogenous parameters and probabilities have values as follows:

H	L	I	M	N	$w(y)$	$p_{G\ B}$	$P(G_1 \mid A_0)$
2000	100	99	98	300	$\log y$	0.5	0.4

Since $P(H \mid G_2) = P(L \mid B_2) = 1$, in the proposed equilibrium, the convertibles will be converted with certainty, provided that $rH/(N + Mr) > 1$. If so, the conversion ratio, r, would satisfy

$$P(H \mid B_0)\frac{Mr}{N + Mr}H + P(L \mid B_0)\frac{Mr}{N + Mr}L = I. \tag{A.2}$$

Solving for r and using the numbers for the exogenous probabilities and parameters given above yields $r \approx 0.2125$. It is straightforward to verify that $rH/(N + Mr) > 1$. Similarly, if equity is issued, the number of new shares, Q, would satisfy

$$P(H \mid A_0)\frac{Q}{N+Q}H + P(L \mid A_0)\frac{Q}{N+Q}L = I. \tag{A.3}$$

This yields $Q \approx 22.3140$.

Next, I state necessary and sufficient conditions for the stated separating equilibrium. (Beliefs are the obvious ones). Prior to each condition, I explain its purpose. Following each condition, I give the numerical value of the left- and right-hand side of the inequality given the assumed values of the exogenous parameters and probabilities. (The left-hand side should always be larger than the right-hand side).

At $t = 2$, call if information is B_2:

$$\begin{aligned} &w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid \phi_0\phi_1 B_2\mathscr{C}\mathscr{D}NC] \\ &\quad \geq w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3) \mid \phi_0\phi_1 B_2\mathscr{C}\mathscr{D}NN]. \end{aligned} \tag{A.4a}$$

$$\text{LHS} = -2.33 \quad \text{and} \quad \text{RHS} = -3.18.$$

At $t = 2$, do not call if information is G_2:

$$\begin{aligned} &w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3) \mid \phi_0\phi_1 G_2\mathscr{C}\mathscr{D}NN] \\ &\quad \geq w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid \phi_0\phi_1 G_2\mathscr{C}\mathscr{D}NC]. \end{aligned} \tag{A.4b}$$

$$\text{LHS} = 3.66 \quad \text{and} \quad \text{RHS} = 0.66.$$

At $t = 1$, call if information is B_1:

$$2w(S_1(\mathscr{C}\mathscr{D}C)) + E[w(S_3) \mid \phi_0 B_1 \mathscr{C}\mathscr{D}C] \geq w(S_1(\mathscr{C}\mathscr{D}N)) + P(G_2 \mid \phi_0 B_1)\{w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3) \mid \phi_0 B_1 G_2 \mathscr{C}\mathscr{D}NN]\} + P(B_2 \mid \phi_0 B_1)\{w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid \phi_0 B_1 B_2 \mathscr{C}\mathscr{D}NC]\}. \quad \text{(A.5a)}$$

LHS = 2.70 and RHS = 2.49.

At $t = 1$, do not call if information is G_1:

$$w(S_1(\mathscr{C}\mathscr{D}N)) + P(G_2 \mid \phi_0 G_1)\{w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3) \mid \phi_0 G_1 G_2 \mathscr{C}\mathscr{D}NN]\} + P(B_2 \mid \phi_0 G_1)\{w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid \phi_0 G_1 \mathscr{C}\mathscr{D}NC]\} \geq 2w(S_1(\mathscr{C}\mathscr{D}C)) + E[w(S_3) \mid \phi_0 G_1 \mathscr{C}\mathscr{D}C]. \quad \text{(A.5b)}$$

LHS = 5.49 and RHS = 4.20.

At $t = 0$, issue convertible debt rather than straight debt if information is B_0:

$$w(S_0(\mathscr{C}\mathscr{D})) + P(B_1 \mid B_0)\{2w(S_1(\mathscr{C}\mathscr{D}C)) + E[w(S_3) \mid B_0 B_1 \mathscr{C}\mathscr{D}C]\} + P(G_1 \mid B_0)\{w(S_1(\mathscr{C}\mathscr{D}N)) + P(G_2 \mid B_0 G_1) \{w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3) \mid B_0 G_1 G_2 \mathscr{C}\mathscr{D}NN]\} + P(B_2 \mid B_0 G_1)\{w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid B_0 G_1 B_2 \mathscr{C}\mathscr{D}NC]\}\} \geq 3w(S_0(\mathscr{S}\mathscr{D})) + E[w(S_3) \mid B_0 \mathscr{S}\mathscr{D}]. \quad \text{(A.6a)}$$

LHS = 5.66 and RHS = 5.50.

At $t = 0$, issue straight debt rather than convertible debt if information is G_0:

$$3w(S_0(\mathscr{S}\mathscr{D})) + E[w(S_3) \mid G_0 \mathscr{S}\mathscr{D}] \geq w(S_0(\mathscr{C}\mathscr{D})) + P(B_1 \mid G_0)\{2w(S_1(\mathscr{C}\mathscr{D}C)) + E[w(S_3) \mid G_0 B_1 \mathscr{C}\mathscr{D}C]\} + P(G_1 \mid G_0)\{w(S_1(\mathscr{C}\mathscr{D}N)) + P(G_2 \mid G_0 G_1) \{w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3) \mid G_0 G_1 G_2 \mathscr{C}\mathscr{D}NN]\} + P(B_2 \mid G_0 G_1) \{w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid G_0 G_1 B_2 \mathscr{C}\mathscr{D}NC]\}\}. \quad \text{(A.6b)}$$

LHS = 7.39 and RHS = 7.05.

At $t = 0$, issue convertible debt rather than equity if information is B_0:

$$\text{LHS (A.6}a\text{)} \geq 3w(S_0(\mathscr{E})) + E[w(S_3) \mid B_0 \mathscr{E}]. \quad \text{(A.6c)}$$

LHS = 5.66 and RHS = 5.55.

At $t = 0$, issue equity rather convertible debt if information is A_0:

$$\begin{aligned}&3w(S_0(\mathscr{E})) + E[w(S_3)\,|\,A_0\mathscr{E}]\\&\quad\geq w(S_0(\mathscr{C}\mathscr{D})) + P(B_1\,|\,A_0)\{2w(S_1(\mathscr{C}\mathscr{D}C)) + E[w(S_3)\,|\,A_0B_1\mathscr{C}\mathscr{D}C]\}\\&\qquad + P(G_1\,|\,A_0)\{w(S_1(\mathscr{C}\mathscr{D}N)) + P(G_2\,|\,A_0G_1)\\&\qquad\{w(S_2(\mathscr{C}\mathscr{D}NN)) + E[w(S_3)\,|\,A_0G_1G_2\mathscr{C}\mathscr{D}NN]\} + P(B_2\,|\,A_0G_1)\\&\qquad\{w(S_2(\mathscr{C}\mathscr{D}NC)) + E[w(S_3)\,|\,A_0G_1B_2\mathscr{C}\mathscr{D}NC]\}\}.\end{aligned} \tag{A.6d}$$

$$\text{LHS} = 5.40 \quad \text{and} \quad \text{RHS} = 5.38.$$

At $t = 0$, issue equity rather than straight debt if information is A_0:

$$3w(S_0(\mathscr{E})) + E[w(S_3)\,|\,A_0\mathscr{E}] \geq 3w(S_0(\mathscr{S}\mathscr{D})) + E[w(S_3)\,|\,A_0\mathscr{S}\mathscr{D}]. \tag{A.6e}$$

$$\text{LHS} = 5.40 \quad \text{and} \quad \text{RHS} = 5.12.$$

At $t = 0$, issue straight debt rather than equity if information is G_0:

$$3w(S_0(\mathscr{S}\mathscr{D})) + E[w(S_3)\,|\,G_0\mathscr{S}\mathscr{D}] \geq 3w(S_0(\mathscr{E})) + E[w(S_3)\,|\,G_0\mathscr{E}]. \tag{A.6f}$$

$$\text{LHS} = 7.39 \quad \text{and} \quad \text{RHS} = 6.30.$$

This concludes the proof by numerical example.

Analytical Proof. Let H, L, N, and $P_{G|B}$ be given. I will show that given $w(y) = \log y$ and the information process specified above, one can find I, M, r, Q, and $P(G_1\,|\,A_0)$ such that equations (A.4)–(A.6) and the financing constraint are satisfied.

Define $x \equiv P_{G|B}$, $\gamma \equiv 2x - x^2$, $z \equiv P_{G_1|A_0}$, and $\lambda \equiv z + x - zx$. γ is the probability of a high payoff given B_0, and λ is the probability of a high payoff given A_0.

I will look for cases where the convertibles are converted with certainty in the proposed separating equilibrium. Thus I make the following conjecture:

Conjecture. $rl_{\mathscr{C}\mathscr{D}} < 1 < rh_{\mathscr{C}\mathscr{D}}$. I will first address the question of what restrictions the conjecture imposes on the parameters. Given the conjecture, (A.2) remains true. This can be rewritten as

$$r = \frac{IN}{M(\gamma H + (1-\gamma)L - I)}. \tag{A.7}$$

Hence, $rl_{\mathscr{C}\mathscr{D}} = (I/M) \times (L/(\gamma H + (1-\gamma)L))$ and $rh_{\mathscr{C}\mathscr{D}} = (I/M) \times (H/(\gamma H + (1-\gamma)L))$, implying that the conjecture is true only if

$$\frac{\gamma H + (1-\gamma)L}{H} < \frac{I}{M} < \frac{\gamma H + (1-\gamma)L}{L}. \tag{A.8}$$

Thus, for the conjecture to be true, I and M must be sufficiently close.

To prove that there are values for the parameters satisfying (A.4) and (A.5) and the conjecture, I appeal to the result from Harris and Raviv (1985) that there are such parameter values of H, L, M, N, r with $H > L > M$ (in Harris and Raviv, there is no cost of investment, I). In particular, they show (Harris and Raviv, 1985 p. 1279) that such values exist if for any $\beta \geq 1$ there is M such that

$$\frac{N}{N+Mr}\frac{L}{L-M} > \beta.$$

Substituting for r using (A.7), this becomes

$$\left(\frac{\gamma H + (1-\gamma)L - I}{\gamma H + (1-\gamma)L}\right)\left(\frac{L}{L-M}\right) > \beta. \tag{A.9}$$

This can clearly be achieved by choosing M sufficiently close to L. It is clear that there are values for M and I such that both (A.8) and (A.9) are satisfied. Thus, the separating equilibrium in the conversion game exists provided M and I are sufficiently close to each other and to L. Next it will be seen that this is also the conditions under which (A.6a) is satisfied.

Under the assumption that $w(y) = \log y$, the assumed information process, and the conjecture, (A.6a) is true if and only if

$$\left(\frac{H(2x-x^2)}{N+Mr} + \frac{L(1-x)^2}{N+Mr}\right)\left(\frac{Hx}{N+Mr} + \frac{L(1-x)}{N+Mr}\right)^{2(1-x)}\left(\frac{H}{N+Mr}\right)^{4x-x^2}$$
$$\times\left(\frac{L}{N+Mr}\right)^{(1-x)^2} \geq \left(\frac{H-I}{N}\right)^{3+2x-x^2}\left(\frac{L-I}{N}\right)^{(1-x)^2}. \tag{A.10}$$

If $I \to L$ from below, RHS(A.10) $\to 0$ while LHS(A.10) $\to c > 0$,[20] where

$$c = \left\{\frac{\gamma(H-L)}{N(\gamma H + (1-\gamma)L)}\right\}^4$$
$$\times\{[H(2x-x^2) + L(1-x)^2][Hx + L(1-x)]^{2(1-x)}H^{4x-x^2}L^{(1-x)^2}\} > 0.$$

Thus (A.9) and (A.10) can be satisfied simultaneously by picking M sufficiently close to I and, in turn, I sufficiently close to L. By (A.8),

[20] It is necessary that $I \to L$ from below because of the assumption that $I < L$ and in order to assure that RHS(A.6a) is defined.

this can be done so that the conjecture is verified. Thus, I am done with (A.4)–(A.6a).

I claim now that $h_{\mathcal{FD}} > h_{\mathcal{CD}}$. From Table I, this is seen to be true if and only if $(H - I)/N > H/(N + Mr)$. Some algebra shows that this, in turn, is true if and only if

$$\frac{H}{N} - \frac{Mr}{N}\left\{\frac{H\gamma}{N + Mr} + \frac{L(1-\gamma)}{N + Mr}\right\} > \frac{H}{N + Mr},$$

which, in turn, is true if and only if $H > H\gamma + L(1 - \gamma)$. The latter condition is true since $\gamma \in (0, 1)$ and $H > L$. Hence, $h_{\mathcal{FD}} > h_{\mathcal{CD}}$.

Under the assumed information process and the conjecture, (A.6b) reduces to

$$4w(h_{\mathcal{FD}}) \geq w(\gamma h_{\mathcal{CD}} + (1 - \gamma)l_{\mathcal{CD}}) + 3w(h_{\mathcal{CD}}).$$

This holds for any parameter values satisfying the conjecture since $h_{\mathcal{FD}} > h_{\mathcal{CD}} > l_{\mathcal{CD}}$. Thus I am done with (A.4)–(A.6b).

Next, I will show that for any values of H, L, M, N, r, Q, and x there exists z such that (A.6c) and (A.6d) hold. Under the conjecture and given $w(y) = \log(y)$ and the assumed information process, (A.6c) and (A.6d) become

$$\begin{aligned}&\left(\frac{1}{N + Mr}\right)^4 (\gamma H + (1-\gamma)L)(xH + (1-x)L)^{2(1-x)}H^{2x}\\ &\quad \geq \left(\frac{1}{N + Q}\right)^4 (\lambda H + (1-\lambda)L)^3\end{aligned} \tag{A.11a}$$

and

$$\begin{aligned}&\left(\frac{1}{N + Mr}\right)^4 (\gamma H + (1-\gamma)L)(xH + (1-x)L)^{2(1-z)}H^{2z}\\ &\quad \leq \left(\frac{1}{N + Q}\right)^4 (\lambda H + (1-\lambda)L)^3.\end{aligned} \tag{A.11b}$$

Substituting (A.2) into (A.3) and rearranging, I obtain

$$Q = N\bigg/\left(\frac{\lambda H + (1-\lambda)L}{\gamma H + (1-\gamma)L}\left(\frac{N}{Mr} + 1\right) - 1\right). \tag{A.12}$$

Thus (A.6c) and (A.6d) become, respectively,

$$(\gamma H + (1-\gamma)L)(xH + (1-x)L)^{2(1-x)}H^{2x} \geq \frac{(\lambda H + (1-\lambda)L + (\gamma-\lambda)(L-H)(Mr/N))^4}{\lambda H + (1-\lambda)L}, \tag{A.13a}$$

and

$$(\gamma H + (1-\gamma)L)(xH + (1-x)L)^{2(1-z)}H^{2z} \leq \frac{(\lambda H + (1-\lambda)L + (\gamma-\lambda)(L-H)(Mr/N)^4}{\lambda H + (1-\lambda)L}. \tag{A.13b}$$

Substituting the expression given for r in (A.7) into (A.13), M drops out. Hence, given I, (A.6c) and (A.6d) are independent of M. One can, therefore, satisfy (A.6c), (A.6d), and the conjecture simultaneously.

Recall from (A.10) that (A.6a) can be ensured if $I \to L$ from below. Thus, by continuity, there is an I such that (A.4)–(A.6d) are satisfied if the following two inequalities hold:

$$(\gamma H + (1-\gamma)L)(xH + (1-x)L)^{2(1-x)}H^{2x} > \frac{\left(\lambda H + (1-\lambda)L - L\dfrac{\gamma-\lambda}{\gamma}\right)^4}{\lambda H + (1-\lambda)L} \tag{A.14a}$$

and

$$(\gamma H + (1-\gamma)L)(xH + (1-x)L)^{2(1-z)}H^{2z} < \frac{\left(\lambda H + (1-\lambda)L - L\dfrac{\gamma-\lambda}{\gamma}\right)^4}{\lambda H + (1-\lambda)L}. \tag{A.14b}$$

(A.14a) corresponds to (A.6c), and (A.14b) corresponds to (A.6c). Notice that given H, L, and x, these two inequalities impose restrictions on z but not on the other parameter values. Let

$$f(z) = \log(\gamma H + (1-\gamma)L) + 2(1-z)\log(xH + (1-x)L) + 2z\log H - 4\log\left(\lambda H + (1-\lambda)L - L\frac{\gamma-\lambda}{\gamma}\right) + \log(\lambda H + (1-\lambda)L)$$

and

$$g(z) = \log(\gamma H + (1-\gamma)L) + 2(1-x)\log(xH + (1-x)L) + 2x\log H$$
$$- 4\log\left(\lambda H + (1-\lambda)L - L\frac{\gamma-\lambda}{\gamma}\right) + \log(\lambda H + (1-\lambda)L).$$

Note that λ is a function of z for both $f(z)$ and $g(z)$.

Now, since $\log(\cdot)$ is concave $f(x) < 0$. Furthermore, $f(0) > 0$. Hence, there exists $z \in (0, x)$ such that $f(z) = 0$. Let

$$z_0 = \sup\{z \in (0, x) \mid f(z) = 0\}.$$

Thus, if $z \in (z_0, x)$, $f(z) < 0$. (A.14b) would then be satisfied.

Now, for all $z < x$, $g(z) > f(z)$. In particular, $g(z_0) > f(z_0) = 0$. Thus, by continuity, there exists $z^* \in (z_0, x)$ such that $g(z^*) > 0$ but $f(z^*) < 0$, and by the definitions of $f(z)$ and $g(z)$, (A.14a) and (A.14b) are satisfied for $z = z^*$. Hence (A.4)–(A.6d), the conjecture, and the financing constraints are satisfied for some 5-tuple $(H^*, L^*, M^*, N^*, I^*)$ and prior probabilities x^* and z^*.

By continuity, for any $\hat{I} > I^*$, there is $\hat{M}$ such that (A.4)–(A.6d), the conjecture, and the financing constraints are satisfied for the 5-tuple $(H^*, L^*, \hat{M}, N^*, \hat{I})$ and prior probabilities x^* and z^*. This will be referred to below when dealing with (A.6f).

Next, I shall show that (A.6e) is true for any (H, L, M, N, I) and λ for which the conjecture is true. From (A.12), it can be seen that since $\gamma > \lambda$, $Q > Mr$. Hence, $h_{\mathfrak{C}}$ is smaller than $h_{\mathfrak{C}\mathfrak{Q}}$, which was demonstrated above to be smaller than $h_{\mathcal{F}\mathfrak{Q}}$, i.e., $h_{\mathfrak{C}} < h_{\mathcal{F}\mathfrak{Q}}$. Given the assumed information process, (A.6e) is true if and only if

$$4w(h_{\mathcal{F}\mathfrak{Q}}) \geq 3w(\lambda h_{\mathfrak{C}} + (1-\lambda)l_{\mathfrak{C}}) + w(h_{\mathfrak{C}}).$$

Since $h_{\mathcal{F}\mathfrak{Q}} > h_{\mathfrak{C}} > l_{\mathfrak{C}}$ and $w(\cdot)$ is strictly increasing, this is true for any (H, L, M, N, I) and λ. Thus I am done with (A.4)–(A.6e).

Finally, I turn to (A.6f). For $w(y) = \log(y)$ and the assumed information process, (A.6f) reduces to

$$\left(\frac{1}{N+Q}\right)^4 (\lambda H + (1-\lambda)L)^3 H^{\lambda} L^{1-\lambda} \geq \left(\frac{H-I}{N}\right)^{3+\lambda}\left(\frac{L-I}{N}\right)^{1-\lambda}. \quad \text{(A.15)}$$

This can be assured by choosing I sufficiently close to L. Given H^*, L^*, N^*, x^*, and z^*, let $\hat{I} \geq I^*$ be such that (A.15) is true. Then there is $\hat{M}$ such that (A.4)–(A.6f), the conjecture, and the financing constraints are satisfied for the 5-tuple (H*, L^*, $\hat{M}$, N^*, $\hat{I}$) and prior probabilities x^* and z^*. (r and Q are determined by (A.2) and (A.3), respectively.)

Consistency. The above demonstrates that the equilibrium is perfect Bayesian (or Bayesian perfect Nash). To prove that it is also a sequential equilibrium, it is necessary to prove that beliefs are consistent. This is nontrivial only for out-of-equilibrium beliefs, which is a concern in the proposed equilibrium under the assumed information process only if there is a call of convertibles at date 2 (a failure to call at date 1 signals G_1, and $P(G_2 | G_1)$ is assumed equal to 1). To prove consistency, notice first that the game at date 2 if there is no call at date 1, is a one shot signalling game of two types (since the probability distribution of $\tilde{\delta}$ is independent of ϕ_0 and ϕ_1 under the assumed information process). It has been shown by Fudenberg and Tirole (1991) that for such games the set of sequential and perfect Bayesian equilibria coincide. Given the assumed information process, the proposed equilibrium is, therefore, sequential. ■

Proof of Proposition 1. $S_0(\mathcal{SD}) = (E[\tilde{\delta} | G_0] - I)/N$, $S_0(\mathcal{CD}) = (E[\tilde{\delta} | B_0] - I)/N$, and $S_0(\mathcal{E}) = (E[\tilde{\delta} | A_0] - I)/N$. The proposition follows by Assumption 2, Eq. (4), Theorem 1, and Theorem 2. ■

Proof of Proposition 2. Immediate from Proposition 1. ■

Proof of Proposition 3. By Lemma 3, in the separating equilibrium, $h_{\mathcal{CD}} > h_{\mathcal{E}}$ and $l_{\mathcal{CD}} > l_{\mathcal{E}}$.

If $\tilde{\phi}_0 = A_0$ and the manager issues equity (as prescribed by the equilibrium), his expected utility is

$$3w(S_0(\mathcal{E})) + P_{H|A_0}w(h_{\mathcal{E}}) + P_{L|A_0}w(l_{\mathcal{E}}).$$

If the manager instead follows a strategy of issuing convertible debt and calling it with certainty at time 1, his expected utility is

$$w(S_0(\mathcal{CD})) + 2w(S_1(\mathcal{CD}C)) + P_{H|A_0}w(h_{\mathcal{CD}}) + P_{L|A_0}w(l_{\mathcal{CD}}).$$

By Proposition 1 and Lemma 3, if $S_1(\mathcal{CD}C) \geq S_0(\mathcal{E})$, the manager would do better by deviating from the equilibrium, which would be a contradiction.

By Harris and Raviv's Theorem 5, it follows that $S_0(\mathcal{E}) > S_2(\mathcal{CD}NC)$ also. ■

Proof of Proposition 4. $S_0(\mathcal{E}) = S_1(\mathcal{E}) = S_2(\mathcal{E})$, and $S_0(\mathcal{SD}) = S_1(\mathcal{SD}) = S_2(\mathcal{SD})$. It follows that $\mathrm{Var}[R_t(\mathcal{E})] = \mathrm{Var}[R_t(\mathcal{SD})] = 0$, $t = 1, 2$. Since the priors are nondegenerate, signalling in the conversion game implies that $\mathrm{Var}[R_t(\mathcal{CD})] > 0$, $t = 1, 2$. ■

Proof of Theorem 3. I start by showing that the intuitive criterion breaks the pooling equilibrium in which the manager never calls at date 1, but always calls at date 2.

Let $q \equiv P(G_2 \mid \mathscr{C}\mathscr{D}NN)$; the probability that investors put on the manager's date 2 information being good if investors observe the out-of-equilibrium action *NoCall.* Let

$$S_2(\mathscr{C}\mathscr{D}NN; q) = q(P_{H|G_2} h_{\mathscr{C}\mathscr{D}} + P_{L|G_2} l_{\mathscr{C}\mathscr{D},n}) + (1 - q)(P_{H|B_2} h_{\mathscr{C}\mathscr{D}} + P_{L|B_2} l_{\mathscr{C}\mathscr{D},n}).$$

Finally, let $S_2^*(\mathscr{C}\mathscr{D}NC)$ denote the pooling equilibrium stock price at date 2 conditional on the convertibles being called.

The intuitive criterion will eliminate the pooling equilibrium if the following two conditions are satisfied.

Condition 1. If the manager's information is B_2 and he does not call, the manager will be worse off compared to what he gets in equilibrium no matter how the lack of calling is interpreted. That is, for any $q \in [0, 1]$,

$$w(S_2(\mathscr{C}\mathscr{D}NN; q)) + E[w(S_3) \mid \mathscr{C}\mathscr{D}NNB_2] < w(S_2^*(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid \mathscr{C}\mathscr{D}NCB_2].$$

Condition 2. If the manager's information is G_2 and he does not call, the manager will be better off compared to what he gets in equilibrium if investors interpret the lack of calling as the manager having good information. That is,

$$w(S_2(\mathscr{C}\mathscr{D}NN; q = 1)) + E[w(S_3) \mid \mathscr{C}\mathscr{D}NNG_2] > w(S_2^*(\mathscr{C}\mathscr{D}NC)) + E[w(S_3) \mid \mathscr{C}\mathscr{D}NCG_2].$$

If Condition 1 is met, the manager will always call if he has bad information since not calling will leave him worse off regardless of investors' beliefs. If Condition 2 is met and the manager has good information, he prefers not calling if this is interpreted as such. Hence, if the two conditions hold, the manager has an incentive to refrain from calling only if he has good information. The intuitive criterion says that this should be reflected in investors' beliefs. In particular, a lack of calling should be interpreted as implying that the manager has good information. Thus if the two conditions hold, beliefs supporting the pooling equilibrium fail the intuitive criterion.

Since $S_2(\mathscr{C}\mathscr{D}NN; q)$ is increasing in q, the first condition is always met if the parameters are chosen to support the separating equilibrium in the conversion game, but the second condition does not necessarily hold.

Suppose now that $P(H \mid G_2) = 1$ and that the other parameters of the model have values for which the separating equilibrium exists. Now if investors interpret a lack of calling in period 2 as the manager's having good information, then $S_2(\mathscr{C}\mathscr{D}NN) = h_{\mathscr{C}\mathscr{D}}$. Since $S_2^*(\mathscr{C}\mathscr{D}NC) = P_{H|\mathscr{C}\mathscr{D}}h_{\mathscr{C}\mathscr{D}} + P_{L|\mathscr{C}\mathscr{D}}l_{\mathscr{C}\mathscr{D}}$, where $P_{H|\mathscr{C}\mathscr{D}}$ and $P_{L|\mathscr{C}\mathscr{D}}$ denote the prior probabilities of a high and low payoff, the second condition is satisfied. Pooling is thus eliminated in period 2.

If also $P(G_2 | G_1) = 1$, it can be shown similarly that pooling in period 1 will be eliminated by the intuitive criterion.

If $P(H | G_2) = 1$ and $P(G_2 | G_1) = 1$; given G_1 or G_2, the period 3 stock price is $h_{\mathscr{C}\mathscr{D}}$ with certainty, independent of the manager's call decisions. Therefore, hybrid equilibria in which the manager calls with certainty if his information is B_t and mixes if it is G_t do not exist. This completes the proof. ■

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